Modeling, Estimation and Control of Traffic Networks

Roberto Horowitz
James Fife Endowed Chair
Chair, Department of Mechanical Engineering
University of California, Berkeley
Traffic Management with Connected and Autonomous Vehicles

“Smart Vehicles”

Faculty
- Roberto Horowitz
- Murat Arcak
- Pravin Varaiya

Grad. Students
- Negar Mehr
- Ruolin Li
- Matt Wright

PATH Res.
- Alex Kurzhanskiy
- Ching-Yao Chan

Collaboration
- Francesco Borrelli
PATH – smart vehicle platooning

Reducing energy consumption

Increasing traffic capacity
Smart vehicle platoons can increase throughput in urban roads (30%-50%) - Varaiya et al.

- Platooning can decrease vehicle headway
- It can also increase saturation flow rates at intersections by 50%
- Roadway capacity can be increased by 50%

Traffic Operating System (TOS)
NSF – CPS (Horowitz, Arccak Varaiya)

Effect of *Smart Vehicles* on the traffic network systems
Smart Vehicle Gradual Deployment

• Can increases in roadway capacity translate into increases in traffic network throughput?

• Vehicles select their routes *selfishly*. 
Does replacing a fraction of vehicles with smart vehicles improve the social delay of the network?

\[ \alpha_l := \frac{f^s_l}{f^s_l + f^r_l} \]

Autonomy fraction on link \( l \)

\( f^r_l \)  
regular vehicle flow

\( f^r_l + f^s_l \)  
mixed vehicle flow
Assume that smart vehicles *increase* road capacity

\[
\mu := \frac{m_l}{M_l}
\]

degree of capacity asymmetry of link \( l \)
Mixed Traffic Delay Characterization

BPR link delay function

\[ e_l(f_l^r, f_l^s) = a_l \left( 1 + \gamma_l \left( \frac{f_l^s}{M_l} + \frac{f_l^r}{m_l} \right)^{\beta_l} \right) \]

\( m_l \): regular vehicle capacity

\( M_l \): smart vehicle capacity
Mixed Traffic User Equilibrium

Social delay as a function of autonomy fraction

In this example, social delay decreases as the fraction of autonomous vehicles increases.

\[ \alpha_l = \frac{f_l^s}{f_l^s + f_l^r} \]

Mehr et al. CDC 2018
Mixed Traffic User Equilibrium

Social delay as a function of autonomy fraction

In this example, social delay does not decrease monotonically as the fraction of autonomous vehicles increases.

Braess’ paradox

Mehr et al. CDC 2018
Homogenous Networks with a Single O/D Pair

Theorem: Given a network $G = (N, L, W)$ with an homogenous degree of capacity asymmetry $\mu$, for any demand $r \geq 0$, we have:

For a fixed $0 \leq \alpha \leq 1$, the social delay $J(f)$ is unique for all equilibrium flow vectors $f$.

The social network delay $J(.)$ is a continuous and non-increasing function of the autonomy fraction $\alpha$.

Increasing the fraction of smart vehicles will enhance network performance when their impact is uniform throughout all roadways.
Traffic Operating System (TOS)
Pricing Traffic Networks with Mixed Autonomy

Use road tolling as a network traffic management scheme
Pricing Traffic Networks with Mixed Autonomy

• Vehicles select their routes *selfishly*.

• User equilibria do not always yield the lowest social delay.

• Use *road tolling* as a network traffic management scheme so that:

  *user equilibria will yield the lowest social delay.*
Differentiating tolling achieves a minimum social delay

**Theorem**: Given a network $G = (N, L, W)$ with an **homogenous** degree of capacity asymmetry $\mu$

Let $f^*$ be the optimal flow vector that achieves the **minimum** social delay $J^*$

$$f^* = \arg \left[ \min_{f} \sum_{p \in P} f_p e_p(f) \right]$$

There exists a differential **tolling scheme** such that all induced Wardrop **cost** equilibria attain the **minimum** social delay $J^*$

Mehr, et al. ACC 2019
Differentiating tolling achieves a minimum social delay

**Theorem:** Given a network $G = (N, L, W)$ with an homogenous degree of capacity asymmetry $\mu$

Let $f^*$ be the optimal flow vector that achieves the minimum social delay $J^*$

$$f^* = \arg \left[ \min_f \sum_{p \in P} f_p e_p(f) \right]$$

**Optimal differentiated tolling scheme**

$$\tau^r_i = (f^r_i + f^s_i) \left( \frac{\partial}{\partial f^r_i} e_l(f^r_i, f^s_i) \right) \bigg|_{f^*_i}$$

$$\tau^s_i = (f^r_i + f^s_i) \left( \frac{\partial}{\partial f^s_i} e_l(f^r_i, f^s_i) \right) \bigg|_{f^*_i}$$
Details: ThB02 **Traffic Control**, Franklin 2

14:50-15:10, Paper ThB02.5

*Pricing Traffic Networks with Mixed Vehicle Autonomy*

*by Negar Mehr and Roberto Horowitz*
Summary

• Traffic systems exhibit very interesting complex behavior

• Sensing is key! – currently not sufficient sensing is available

• We can deploy sophisticated traffic estimation and management techniques

• Smart (autonomous and connected) vehicles can make traffic network management even more challenging

• Pricing can be an effective traffic management technique
Acknowledgements

My research presented here has been supported by

- CALTRANS
- National Science Foundation (NSF)
- California PATH